Relativity

The Winning Essay for the Eugene Higgins Five Thousand Dollar Prize

By "Zodiaque" (L. Bolton, London, England)

The reader is probably acquainted with the method of specifying positions of points in a plane by their distances from two mutually perpendicular lines, or if the points are in space by their distances from three mutually perpendicular planes like adjacent sides of a flat-sided box. The method is in fact in common use for exhibiting relations between quantities by graphs or diagrams. These sets of axes, as they are called, together with any scales used for measuring, must be supposed rigid, otherwise the events or points which they are used to specify are indefinite. The lengths which locate any point with reference to a set of axes are called its coördinates.

When such systems are used for physical purposes, they must be supplemented by clocks to enable the times at which events occur to be determined. The clocks must be synchronized, and must go at the same rate, but it must suffice here to state that this is possible without indicating how these conditions can be attained. A system of axes with its clocks will hereinafter be called a Frame of Reference, and every observer will be supposed to be provided with such a frame partaking of his motion. All the objects which partake of an observer's motion will be called his *system*.

It is a question whether among all possible frames of reference any one frame or class of frames is more suited than another for the mathematical statement of physical laws. This is for experience to decide, and a Principle of Relativity is a statement embodying the answer.

The Mechanical Principle of Relativity

It has been ascertained that all such frames are equally suitable for the mathematical statement of general *mechanical* laws, provided that their motion is rectilinear and uniform and without rotation. This fact is comprehended in the general statement that *all unaccelerated frames of reference are equivalent for the statement of the general laws of mechanics.* This is the mechanical principle of relativity.

It is well recognized however that the laws of dynamics as hitherto stated involve the following assumptions:

(1) Lengths of rigid bodies are unaffected by the motion of the frame of reference.
(2) Measured times are likewise unaffected.

That is to say that any length measured on his own system by either of two relatively moving observers appears the same to both observers, or that lengths of objects and rates of clocks do not alter whatever the motion relative to an observer. These assumptions seem so obvious that it is scarcely perceived that they are assumptions at all. Yet this is the case, and as a matter of fact they are both untrue.

The Special Principle of Relativity

Although all unaccelerated frames of reference are equivalent for the purposes of mechanical laws, this is not the case for physical laws generally as long as the above suppositions are adhered to. Electromagnetic laws *do* alter their form according to the motion of the frame of reference; that is to say, if these suppositions are true, electromagnetic agencies act in different ways according to the motion of the system in which they occur. There is nothing *a priori* impossible in this, but it does not agree with experiment. The motion of each locality on the earth is continually changing from hour to hour but no corresponding changes occur in electromagnetic actions. It has however been ascertained that on discarding these suppositions the difficulty disappears, and electromagnetic laws retain their form under all circumstances of unaccelerated motion. According to the theory of relativity, the correct view which replaces these suppositions is deducible from the following postulates:

(1) By no experiment conducted on his own system can an observer detect the unaccelerated motion of his system.
(2) The measure of the velocity of light *in vacuo* is unaffected by relative motion between the observer and the source of light.

Both these postulates are well established by experiment. The first may be illustrated by the familiar difficulty of determining whether a slowly moving train one happens to be sitting in, or an adjacent one, is in motion. The passenger has either to wait for bumps (that is, accelerations) or else he has to look out at some adjacent objects which he knows to be fixed, such as a building (that is, he has to perform an experiment on something outside his system), before he can decide.

The second postulate is an obvious consequence of the wave theory of light. Just as waves in water, once started by a ship, travel through the water with a velocity independent of the ship, so waves in space travel onward with a speed bearing no relation to that of the body which originated them. The statement however is based on experiment, and can be proved independently of any theory of light.

It is not difficult to deduce from these postulates certain remarkable conclusions relating to the systems of two observers, A and B, in relative motion, among them the following:

(1) Objects on B's system appear to A to be shorter in the direction of relative motion than they appear to B.

(2) This opinion is reciprocal. B thinks that A's measurements on A's system are too great.

(3) Similarly for times: each observer thinks that the other's clocks have a slower rate than his own, so that B's durations of time appear shorter to B than to A, and conversely.

(4) Events which appear simultaneous to A do not in general appear so to B, and conversely.

(5) Lengths at right angles to the direction of motion are unaffected.

(6) These effects vary with the ratio of the relative velocity to that of light. The greater the relative velocity, the greater the effects. They vanish if there is no relative velocity.

(7) For ordinary velocities the effects are so small as to escape notice. The remarkable point however is their occurrence rather than their magnitude.

(8) The observers similarly form different estimates of the velocities of bodies on each other's systems. The velocity of light however appears the same to all observers.

Taking into account these revised views of lengths and times the mechanical principle of relativity may be extended to physical laws generally as follows: *All unaccelerated frames of reference are equivalent for the statement of the general laws of physics.* In this form the statement is called the Special, or Restricted, Principle of Relativity, because it is restricted to unaccelerated frames of reference. Naturally the laws of classical mechanics now require some modification, since the suppositions of unalterable lengths and times no longer apply.

The Four Dimensional Continuum

Lengths and times therefore have not the absolute character formerly attributed to them. As they present themselves to us they are relations between the object and the observer which change as their motion relative to him changes. Time can no longer be regarded as something independent of position and motion, and the question is what is the reality? The only possible answer is that objects must be regarded as existing in four dimensions, three of these being the ordinary ones of length, breadth and thickness, and the fourth, time. The term "space" is applicable only by analogy to such a region; it has been called a "continuum," and the analogue of a point in ordinary three-dimensional space has been appropriately called an "event." By "dimension" must be understood merely one of four independent quantities which locate an event in this continuum. In the nature of the case any clear mental picture of such a continuum is impossible; mankind does not possess the requisite faculties. In this respect the mathematician enjoys a great advantage. Not that he can picture the thing mentally any better than other people, but his symbols enable him to abstract the relevant properties from it and to express them in a form suitable for exact treatment without the necessity of picturing anything, or troubling whether or not the properties are those on which others rely for their conceptions.

Gravitation and Acceleration

The limitation of statements of general law to uniformly moving systems is hardly satisfactory. The very concept of general law is opposed to the notion of limitation. But the difficulties of formulating a law so that the statement of it shall hold good for all observers, whose systems may be moving with different and possibly variable accelerations, are very great. Accelerations imply forces which might be expected to upset the formulation of any general dynamical principles, and besides, the behavior of measuring rods and clocks would be so erratic as to render unmeaning such terms as rigidity and measured time, and therefore to preclude the use of rigid scales, or of a rigid frame of reference which is the basis of the foregoing investigation.

The following example taken from Einstein will make this clear, and also indicate a way out of the difficulty. A rotating system is chosen, but since rotation is only a particular case of acceleration it will serve as an example of the method of treating accelerated systems generally. Moreover, as it will be seen, the attribution of acceleration to the system is simply a piece of scaffolding which can be discarded when the general theory has been further developed.

Let us note the experiences of an observer on a rotating disk which is isolated so that the observer has no direct means of perceiving the rotation. He will therefore refer all the occurrences on the disk to a frame of reference fixed with respect to it, and partaking of its motion.

He will notice as he walks about on the disk that he himself and all the objects on it, whatever their constitution or state, are acted upon by a force directed away from a certain point upon it and increasing with the distance from that point. This point is actually the center of rotation, though the observer does not recognize it as such. The space on the disk in fact presents the characteristic properties of a gravitational field. The force differs from gravity as we know it by the fact that it is directed away from instead of toward a center, and it obeys a different law of distance, but this does not affect the characteristic properties that it acts on all bodies alike, and cannot be screened from one body by the interposition of another. An observer aware of the rotation of the disk would say that the force was centrifugal force; that is, the force due to inertia which a body always exerts when it is accelerated.

Next suppose the observer to stand at the point of the disk where he feels no force, and to watch someone else comparing, by repeated applications of a small measuring rod, the circumference of a circle having its center at that point, with its diameter. The measuring rod when laid along the circumference is moving lengthwise relatively to the observer, and is therefore subject to contraction by his reckoning. When laid radially to measure the diameter this contraction does not occur. The rod will therefore require a greater proportional number of applications to the circumference than to the diameter, and the number representing the ratio of the circumference of the circle to the diameter thus measured will therefore be greater than 3.14159+, which is its normal value. Moreover the relative velocity decreases as the center is approached, so that the contraction of the measuring rod is less when applied to a smaller circle; and the ratio of the circumference to the diameter, while still greater than the normal, will be nearer to it than before, and the smaller the circle the less the difference from the normal. For circles whose centers are not at the point of zero force the confusion is still greater, since the velocities relative to the observer of points on them now change from point to point. The whole scheme of geometry as we know it is thus disorganized. Rigidity becomes an unmeaning term since the standards by which alone rigidity can be tested are themselves subject to alteration. These facts are expressed by the statement that the observer's measured space is non-Euclidean; that is to say, in the region under consideration measurements do not conform to the system of Euclid.

The same confusion arises in regard to clocks. No two clocks will in general go at the same rate, and the same clock will alter its rate when moved about.

The General Principle of Relativity

The region therefore requires a space-time geometry of its own, and be it noted that with this special geometry is associated a definite gravitational field, and if the gravitational field ceases to exist, for example if the disk were brought to rest, all the irregularities of measurement disappear, and the geometry of the region becomes Euclidean. This particular case illustrates the following propositions which form the basis of this part of the theory of relativity:

(1) Associated with every gravitational field is a system of geometry, that is, a structure of measured space peculiar to that field.

(2) Inertial mass and gravitational mass are one and the same.

(3) Since in such regions ordinary methods of measurement fail, owing to the indefiniteness of the standards, the systems of geometry must be independent of any particular measurements.

(4) The geometry of space in which no gravitational field exists is Euclidean.

The connection between a gravitational field and its appropriate geometry suggested by a case in which acceleration was their common cause is thus assumed to exist from whatever cause the gravitational field arises. This of course is pure hypothesis, to be tested by experimental trial of the results derived therefrom.

Gravitational fields arise in the presence of matter. Matter is therefore presumed to be accompanied by a special geometry, as though it imparted some peculiar kink or twist to space which renders the methods of Euclid inapplicable, or rather we should say that the geometry of Euclid is the particular form which the more general geometry assumes when matter is either absent or so remote as to have no influence. The dropping of the notion of acceleration is after all not a very violent change in point of view, since under any circumstances the observer is supposed to be unaware of the acceleration. All that he is aware of is that a gravitational field and his geometry coexist.

The prospect of constructing a system of geometry which does not depend upon measurement may not at first sight seem hopeful. Nevertheless this has been done. The system consists in defining points not by their distances from lines or planes (for this would involve measurement) but by assigning to them arbitrary numbers which serve as labels bearing no relation to measured distances, very much as a house is located in a town by its number and street. If this labeling be done systematically, regard being had to the condition that the label-numbers of points which are close together should differ from one another by infinitesimal amounts only, it has been found that a system of geometry can actually be worked out. Perhaps this will appear less artificial when the fact is called to mind that even when standards of length are available no more can be done to render lengths of objects amenable to calculation than to assign numbers to them, and this is precisely what is done in the present case. This system of labeling goes by the name of "Gaussian coördinates" after the mathematician Gauss who proposed it.

It is in terms of Gaussian coördinates that physical laws must be formulated if they are to have their widest generality, and the general principle of relativity is that *all Gaussian systems are equivalent for the statement of general physical laws.* For this purpose the labeling process is applied not to ordinary space but to the four dimensional space-time continuum. The concept is somewhat difficult and it may easily be aggravated into impossibility by anyone who thinks that he is expected to visualize it. Fortunately this is not necessary; it is merely one of these irrelevancies to which those who are unaccustomed to think in symbols are liable.

It will now be seen that among physical laws the law of gravitation stands pre-eminent, for it is gravitating matter which determines the geometry, and the geometry determines the form of every other law. The connection between the geometry and gravitation is the law of gravitation. This law has been worked out, with the result that Newton's law of the inverse square is found to be approximate only, but so closely approximate as to account for nearly all the motions of the heavenly bodies within the limits of observation. It has already been seen that departure from the Euclidean system is intensified by rapidity of motion, and the movements of these bodies are usually too slow for this departure to be manifest. In the case of the planet Mercury the motion is sufficiently rapid, and an irregularity in its motion which long puzzled astronomers has been explained by the more general law.

Another deduction is that light is subject to gravitation. This has given rise to two predictions, one of which has been verified. The verification of the other is as yet uncertain, though the extreme difficulty of the necessary observations may account for this.

Since light is subject to gravitation it follows that the constancy of the velocity of light assumed in the earlier part of this paper does not obtain in a gravitational field. There is really no inconsistency. The velocity of light is constant in the absence of gravitation, a condition which unaccelerated motion implies. The special principle of relativity is therefore a limiting case of the general principle.

Some Einstein Contest Personalities

By the Einstein Editor

It is appropriate at this time to say a word in regard to Mr. Eugene Higgins, the donor of the splendid prize of $5,000 for the best essay on the Einstein Theory of Relativity, which was announced for the first time in this paper some months ago.

Mr. Higgins is a graduate of Columbia University; was brought up in New York and lived here for many years. He is a bachelor, with the freedom that all that implies; and with no special ties to keep him in this country he has traveled extensively abroad. He has a handsome residence on the banks of the Seine, in the city of Paris, and makes occasional trips to this country to look after his interests here. He is a gentleman of refinement and culture, interested in all that pertains to intellectual life, and particularly to physics and mathematics. It is his interest in this particular line of work which has prompted him to offer this magnificent prize for the best essay on the subject of Relativity.

It has never been announced, but perhaps it is appropriate at this time to state that it was Mr. Higgins who some years ago offered a prize of $500 for a mathematical essay which was published in the SCIENTIFIC AMERICAN at that time. It was his wish that his name should not in any way be connected with the last mentioned gift to the development of science. This is in entire harmony with his character, which is one of self-effacement; it was only after considerable persuasion that he allowed his name to be used in connection with the present prize. It goes without saying that he has absolutely no ulterior motive beyond his desire for the advancement of scientific knowledge in general.

It is hardly necessary for us to mention to our readers the fact that these prizes have been offered by Mr. Higgins without any prompting or suggestion on our part. It has been thought by the Editor that it would not be proper for us to make this announcement, however, without having these facts thoroughly understood by our readers. We venture to follow this attitude without the knowledge of Mr. Higgins, with whom we have no opportunity of communicating as to his wishes except by cable.

We regret that at the present time it is not possible for us to supplement this information about the donor of the prize with a similar statement about the winner. Mr. Bolton, we suppose, may fairly be called unknown in a strictly scientific sense, though he is a professional man of distinction in his field. He is on the staff of the British Patent Office, in a position which we are unable to define exactly at this writing, but which is one of rank. It will be recalled that Einstein himself was in the Swiss Patent Office for some years.

That Mr. Bolton did not take the prize through default of serious competition will be evident from a brief mention of a few of his most distinguished competitors. Dr. William H. Pickering of the Harvard Observatory in Jamaica; our own Dr. Russell, Royal Astronomical Society medalist for the year; and Dr. William de Sitter, the distinguished Netherlander, are among the astronomers of note who took part in the contest. Other continental competitors were Schlick, author of "Space and Time in Contemporary Physics," and Becquerel, who should need no word of introduction. Perhaps the most distinguished British name which was given up by any of the little sealed envelopes was that of H. H. Turner, of Oxford; others were Dr. E. N. da C. Andrade, Professor of Physics in the Ordnance College at Woolwich, and Dr. T. Royds, of the Kodaikanal Observatory in southern India. Among American physicists we find the names of Prof. H. F. Moore, of the University of Illinois; Dr. J. S. Ames, of Johns Hopkins; Dr. W. F. Swann, of the University of Minnesota and Dr. A. G. Webster, of Clark University. Dr. G. D. Birkhoff, of Harvard, we should say heads the list of mathematicians pure and simple who competed; and we must list the well known meteorologist, Dr. Alexander McAdie, of Harvard, an occasional contributor to our columns. That even the men whose primary interest lies in the severely practical direction are not strangers to the intricacies of abstract theory is indicated by the entry of Dr. C. E. K. Mees of the giant commercial photographic laboratory at Rochester. In short, Mr. Bolton has come out at the top of a very distinguished company.

In their work of gradual elimination of those essays which were not the best, the Einstein Judges found that by all means the most effective test to apply was that which arises from the fact that when a man writes about the Einstein theories in 3,000 words, the most momentous problem confronting him is what to leave out. Examination of the essays brought to light without much difficulty about twenty that stood out well above the others in this regard. Mr. Bolton's winning essay is the example par excellence of this merit of *advantageous selection. Everybody will of course agree that he says admirably what he has to say; but the real reason why his essay was ultimately chosen over its most pressing rivals was the extraordinarily fine judgment which he used in deciding just what he would say and what he would leave unsaid. We do not believe it would be possible to make any material improvement upon Mr. Bolton's selection of the ground to be covered in an essay of this character.*— THE EINSTEIN EDITOR.

We, the Judges in the Einstein Prize Essay Contest, hereby state that it is our united judgment that the essay "Relativity," submitted under the nom de plume "Zodiaque," is the best essay received within a proper interpretation of the conditions and aims of the contest; and we award to its author the prize of $5,000 offered by Mr. Eugene Higgins of Paris.
Leigh Page
E.P. Adams

Alike from the nature of the subject, from the fact that Dr. Eddington has written a rather bulky book about it, and for that matter from our remarks on the opposite page, it should be clear to the reader that Mr. Bolton has by no means exhausted or attempted to exhaust his subject. A few of the very best of the competing essays we shall print in full in subsequent issues of the SCIENTIFIC AMERICAN *and the* SCIENTIFIC AMERICAN MONTHLY; *but a few only. We shall, on the other hand, print portions of and extracts from a comparatively large number. The aim will be to set before our readers everything of real value which the contest has brought out; but to use no more space in duplication than is unavoidable, and to print nothing that requires any serious editorial questioning. Many of the excerpts which will thus appear will cover points which Mr. Bolton leaves untouched; many will develop more fully or to better advantage points on which he leaves something to be desired in either of these respects. We shall emphasize at all times that Mr. Bolton's essay is not the only thing of value that has come out of the contest, and that no reader can do justice to the relativity theories by reading it and ignoring the others. In publishing other essays or parts of essays, and even in calling specific attention to the points in which they add to or improve upon Mr. Bolton's essay, we are in no sense criticising the latter, or saying anything on which a just complaint against the award of the judges may be based.*—THE EINSTEIN EDITOR.